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Time-variant reliability analysis of three-dimensional slopes based on Support Vector Machine method

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Abstract: In the reliability analysis of slope, the performance functions derived from the most available stability analysis procedures of slopes are usually implicit and cannot be solved by first-order second-moment approach. A new reliability analysis approach was presented based on three-dimensional Morgenstern-Price method to investigate three-dimensional effect of landslide in stability analyses. To obtain the reliability index, Support Vector Machine (SVM) was applied to approximate the performance function. The time-consuming of this approach is only 0.028% of that using Monte-Carlo method at the same computation accuracy. Also, the influence of time effect of shearing strength parameters of slope soils on the long-term reliability of three-dimensional slopes was investigated by this new approach. It is found that the reliability index of the slope would decrease by 52.54% and the failure probability would increase from 0.000 705% to 1.966%. In the end, the impact of variation coefficients of c and f on reliability index of slopes was taken into discussion and the changing trend was observed.

Key words: slope engineering; Morgenstern-Price method; three dimension; Support Vector Machine; time-variant reliability

1 Introduction

The establishment of limit state equation for slope reliability analyses is based on the calculation of factor of safety of the slope [1] which can be obtained usually by two-dimensional limit equilibrium analysis methods. Generally, two-dimensional limit equilibrium analysis method can meet the engineering requirement. However, in engineering practice, landslide mass is a spatial assemblage of a variety of rock and soil masses and subjected to asymmetric external forces, also its failure surface is of complicated geometry. It should be more reliable to analyze and evaluate the slope stability in three dimensions. Hence, based on two-dimensional limit equilibrium slice method, many researchers proposed a number of analysis methods of three-dimensional slopes upon different assumptions [2−5]. Afterwards, the three-dimensional Morgenstern-Price method (3D M-P method for short) was developed to account for both the static equilibrium in three directions and the moment equilibrium about the sliding principal axis [5]. With brief computation formula in the 3D M-P method, a three-dimensional factor of safety, which is of stable convergence, can be obtained quickly by simple iterations rather than solving for the enormous equations. Such merits bring in its wide application in practical engineering.

There have been many probes on the two-dimensional reliability analysis of the slope [6−10], but a few have devoted into the reliability of three-dimensional limit equilibrium methods for the slope stability [11−14]. Three-dimensional simplified Bishop Method is applied to build the mechanic model and Monte-Carlo method is used to form random sample of geotechnical parameters to solve for the reliability index of the three-dimensional slope. Then, the influence of both the variation and various probability distribution forms of strength parameters on the reliability index is discussed [11]. But its mechanic model only accounts for the static equilibrium in two directions which is excessively simplified and Monte-Carlo method is deficient for its low computation efficiency despite of its capacity to acquire enough accuracy. Spatial random field theory is imposed into three-dimensional simplified Bishop Method. The reliability problem of anisotropic soil slopes is solved by partial average approach and it can make spatial prediction of the slope by computing its maximum failure probability [12−13]. But, it is not consistent to the real situation by assuming an identical

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mean value for c and φ. Finite Element Method is adopted to build the three-dimensional slope model and follow the solution of the reliability index via Monte-Carlo method [14]. Whereas, there are a number of factors included in the FEM model, such as the selection of constitutive model to simulate the soil, the meshing form and calculation arguments. Moreover, the established three-dimensional model is relatively complex such that the time-consuming of Monte-Carlo method to solve for reliability values is intolerable. Hence, a new calculation approach with superior efficiency was developed based on 3D M-P method to analyze the reliability of the three-dimensional slope herein.

As the formula of the stability factor of safety for 3D M-P method is a highly complex implicit recursive expression, it cannot be achieved to establish an explicit limit state equation in conventional ways when making reliability analysis. For two-dimensional slopes, investigators take advantage of response surface quadratic function method [15], neural network [16], and Support Vector Machine [9] to approximate the limit state equation, respectively. The quadratic polynomial (response surface method) has a limited capacity to approximate the nonlinear functions, and neural network is of high probability to trap into local minima. As a result, the computation efficiency and accuracy can be influenced very easily. Therefore, the small sample technique in statistical learning theory is introduced to investigate the calculation method of reliability for three-dimensional slopes based on Support Vector Machine (SVM).

Moreover, the rock-soil masses on the earth surface experience a chronic weathering. This results in the varying values of shear strength parameters, along with the increase of weathering depth over time [17] which can be denoted that the shear strength parameters of slopes have time effect. Utilizing the formula of shear strength parameters of soil mass against time in Ref.[18], we analyze the time-variant reliability of three-dimensional slopes in association with discussion about the variation of reliability versus variation coefficients in case study.

2 Introduction of 3D M-P method for slope stability

2.1 Basic assumptions

The selection of coordinates and column partition are illustrated in Fig.1 and the free-body diagram of a single column is shown in Fig.2. Assumptions are presented as follows [5]:

1) To a specific column (i, j), the intercolumnar shearing force $V_{(i,j)}$ and normal force $E_{(i,j)}$ exerting on row interfaces (surfaces *ABFE* and *DCGH*, parallel to surface *YOZ*, as shown in Fig.2) meet the following relationship:

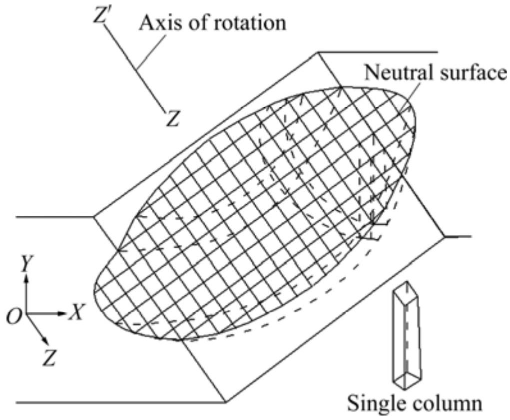

Fig.1 Column partition of landslide mass

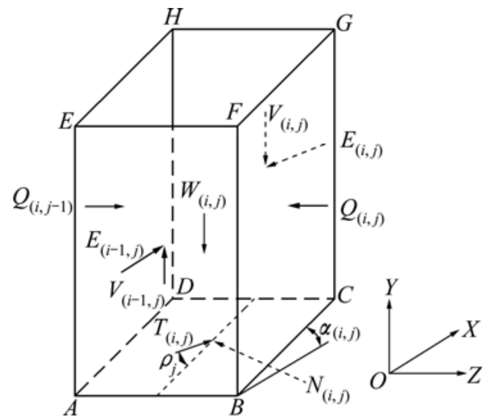

Fig.2 Free-body diagram of column

$$V_{(i,j)} = \lambda_j f_{(i,j)} E_{(i,j)} \tag{1}$$

where λ_j is the scale factor; $f_{(i,j)}$ is the intercolumnar force function. This assumption is similar to that for two-dimensional Morgenstern-Price method.

2) Force $Q_{(i,j)}$ acts on column interfaces (surfaces *ADHE* and *BCGH*, parallel to surface *XOY*, in Fig.2) in the horizontal direction which is parallel to axis Z.

3) The angle between the shearing force $T_{(i,j)}$ acting on the bottom slipping surface and surface *XOY* is of magnitude of ρ_j. Note that ρ_j is positive when the component of shearing force directs in the positive Z.

4) Assume that columns in a specific column (Z is a constant) have an identical ρ_j, otherwise, to different values of coordinate Z, the distribution form of ρ_j is given by

$$\begin{cases} \rho_{Rj} = \kappa z, \ z \ge 0 \\ \rho_{Lj} = \eta\kappa z, \ z < 0 \end{cases} \tag{2}$$

2.2 Computation flow of factors of safety for 3D M-P method

For 3D M-P method, factors of safety of the slope can be derived in steps as follows:

1) When the centric of rotation, long and short axes and mechanical strength parameters (e.g. c and φ) are specified, the landslide mass is divided into columns in L rows and M columns automatically;

2) For the determinate initial factors of safety of the three-dimensional slope as $F_{3S}^0, \lambda_0, \rho_0$ or η_0, κ_0, calculate the gravities of all the columns $W_{(i,j)}$;

3) Solve Eqs.(4)−(8), and substituting the values into Eq.(3) yields the value of F_{3S}:

$$F_{3S}=\frac{\sum_{j=1}^{M}\left(\sum_{i=1}^{L-1}R_{(i,j)}\prod_{k=i}^{L-1}\psi_{(k,j)}+R_{(L,j)}\right)}{\sum_{j=1}^{M}\left(\sum_{i=1}^{L-1}P_{(i,j)}\prod_{k=i}^{L-1}\psi_{(k,j)}+P_{(L,j)}\right)} \tag{3}$$

$$R_{(i,j)}=W_{(i,j)}m_{X(i,j)}-(m_{X(i,j)}n_{Y(i,j)}-m_{Y(i,j)}n_{X(i,j)})\cdot u_{(i,j)}A_{(i,j)}\tan\varphi_{(i,j)}+(m_{X(i,j)}n_{Y(i,j)}-m_{Y(i,j)}n_{X(i,j)})C_{(i,j)}A_{(i,j)} \tag{4}$$

$$P_{(i,j)}=-W_{(i,j)}n_{X(i,j)} \tag{5}$$

$$\phi_{(i,j)}=(\lambda_j f_{(i,j)}m_{X(i,j)}-m_{Y(i,j)})\tan\varphi_{(i,j)}+(\lambda_j f_{(i,j)}n_{X(i,j)}-n_{Y(i,j)})F_{3S} \tag{6}$$

$$\phi_{(i-1,j)}=(\lambda_j f_{(i-1,j)}m_{X(i-1,j)}-m_{Y(i-1,j)})\tan\varphi_{(i-1,j)}+(\lambda_j f_{(i-1,j)}n_{X(i-1,j)}-n_{Y(i-1,j)})F_{3S} \tag{7}$$

$$\psi_{(i-1,j)}=[(\lambda_j f_{(i-1,j)}m_{X(i,j)}-m_{Y(i,j)})\tan\varphi_{(i,j)}+(f_{(i-1,j)}n_{X(i,j)}-n_{Y(i,j)})F_{3S}]/\phi_{(i-1,j)} \tag{8}$$

where $u_{(i,j)}$ is the pore water pressure; $A_{(i,j)}$ is the area of the column base; $C_{(i,j)}$ is the cohesion intercept; $f_{(i,j)}$ is the factor of safety of the j-th row; $W_{(i,j)}$ is the gravity of the column; $n_{X(i,j)}$, $n_{Y(i,j)}$ and $n_{Z(i,j)}$ are directional derivatives of the normal lines of slipping surface; $m_{X(i,j)}$, $m_{Y(i,j)}$ and $m_{Z(i,j)}$ are directional derivatives of the shearing force $T_{(i,j)}$.

4) Apply the obtained F_{3S} and the end conditions: $E_{(0,j)}$=0, $E_{(L,j)}$=0 into Eq.(9) to determine $E_{(i,j)}$ (i=1, 2, ···, L; j=1, 2, ···, M):

$$E_{(i,j)}\phi_{(i,j)}=\psi_{(i-1,j)}E_{(i-1,j)}\phi_{(i-1,j)}-F_{3S}P_{(i,j)}+R_{(i,j)} \tag{9}$$

5) Use the obtained F_{3S} and $E_{(i,j)}$ into Eq.(10) to determine the normal stress $N_{(i,j)}$ exerting on the base of column:

$$N_{(i,j)}=\frac{u_{(i,j)}A_{(i,j)}\tan\varphi_{(i,j)}-C_{(i,j)}A_{(i,j)}-F_{3S}(E_{(i-1,j)}-E_{(i,j)})}{n_{x(i,j)}F_{3S}+m_{x(i,j)}\tan\varphi_{(i,j)}} \tag{10}$$

6) If ρ in Eq.(2) is a constant, substituting the values of F_{3S} and $N_{(i,j)}$ into Eqs.(11)−(12), we obtain the value of ρ:

$$m_{Z(i,j)}=m_{Zj}=m_Z=-F_{3S}\frac{\sum_{j=1}^{M}\sum_{i=1}^{L}N_{(i,j)}n_{Z(i,j)}}{\sum_{j=1}^{M}\sum_{i=1}^{L}\left[(N_{(i,j)}-u_{(i,j)}A_{(i,j)})\tan\varphi_{(i,j)}+C_{(i,j)}A_{(i,j)}\right]} \tag{11}$$

$$\rho=\sin^{-1}(m_Z) \tag{12}$$

7) If ρ in Eq.(2) is not a constant, using the values of F_{3S} and $N_{(i,j)}$ in Eq.(13)−(14), we have the values of η and κ:

$$\kappa=\sum_{j=M_1}^{M}\sin^{-1}\left\{-F_{3S}\sum_{i=1}^{L}N_{(i,j)}n_{Z(i,j)}/\sum_{i=1}^{L}[(N_{(i,j)}-u_{(i,j)}A_{(i,j)})\tan\varphi_{(i,j)}+C_{(i,j)}A_{(i,j)}]\right\}/\sum_{j=M}^{M}Z_{Rj} \tag{13}$$

$$\eta=\sum_{j=M_1}^{M_1}\sin^{-1}\left\{-F_{3S}\sum_{i=1}^{L}N_{(i,j)}n_{Z(i,j)}/\sum_{i=1}^{L}[(N_{(i,j)}-u_{(i,j)}A_{(i,j)}\tan\varphi_{(i,j)}+C_{(i,j)}A_{(i,j)}\right\}/\kappa\sum_{j=1}^{M_1}Z_{Lj} \tag{14}$$

8) Substituting the value of $E_{(i,j)}$ obtained in Eq.(4) into Eq.(15) , we have

$$\lambda=\frac{\sum_{j=1}^{M}\sum_{i=1}^{L}d_{(i,j)}(E_{(i,j)}+E_{(i-1,j)})}{\sum_{j=1}^{M}\sum_{i=1}^{L}b_{(i,j)}(f_{(i,j)}E_{(i,j)}+f_{(i-1,j)}E_{(i-1,j)})} \tag{15}$$

where $d_{(i,j)}$ is the average projection length of the base of the (i, j)-th column in the direction of axis Y and $b_{(i,j)}$ in the direction of axis X (see Fig.3).

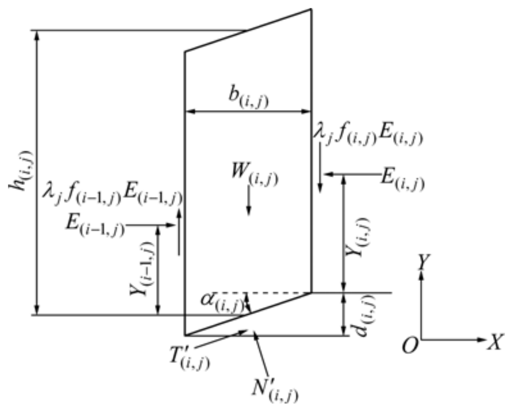

Fig.3 Schematic diagram of plane forces exerting on single column

9) Check if they meet the requirements as follows: $\left|F_{3S}-F_{3S}^0\right|\le\varepsilon_1$, $|\lambda-\lambda_0|\le\varepsilon_2$ and $|\rho-\rho_0|\le\varepsilon_3$ or $[(\eta-\eta_0)^2+(\kappa-\kappa_0)^2]^{1/2}\le\varepsilon_3$ (ε_1, ε_2 and ε_3 are the prescribed calculation

accuracies). If they do, then the obtained F_{3S}, λ, ρ or η and κ are the solutions; otherwise, let $F_{3S}^0=F_{3S}$, $\lambda_0=\lambda$ and $\rho=\rho_0$ or $\eta=\eta_0$, $\kappa=\kappa_0$ return to step 3) to recalculate till they meet the requirements.

3 Reliability analyses for three-dimensional slopes based on SVM

3.1 Optimal fitting algorithm of SVM

SVM theory based on the small sample support was derived from the statistically learning method proposed by VAPNIK et al. The basic principle of the algorithm can consult Ref.[19].

The radial basis kernel function is a widespread kernel function and its corresponding feature space is infinite-dimensional in which a limited number of data samples are linearly separable. Hence, applying radial basis kernel function to the expression of SVM nonlinear fitting function, we have

$$f(\boldsymbol{x})=\sum_{i=1}^{n}(\alpha_i-\alpha_i^*)\exp(-\gamma_i\,|\,\boldsymbol{x}-\boldsymbol{x}_i\,|^{\,2})+b \tag{16}$$

where γ_i is a constant on which the width of function around the center depends.

3.2 Determination method of sample points and solution details of reliability

Assuming that there are a set of parameters with greater uncertainties among basic parameters which influence the slope reliability, we denote them as $\boldsymbol{X}=(x_1, x_2, \cdots, x_d)$, the corresponding means as $\boldsymbol{\mu}=(\mu, \mu_2, \cdots, \mu_d)$, the mean square error as $\boldsymbol{\sigma}=(\sigma_1, \sigma_2, \cdots, \sigma_d)$ by analogy. According to the design method of orthogonal test and '3σ' principle in statistical theory, we start to sample the basic random variables. For the first sample, we let it be $\boldsymbol{X}_0=(\mu_1, \mu_2, \cdots, \mu_d)$, then continue the sampling such that $\boldsymbol{X}_1=(\mu_1+f\sigma_1, \mu_2, \cdots, \mu_d)$, $\cdots$, $\boldsymbol{X}_j=(\mu_1, \mu_2, \cdots, \mu_j+f\sigma_j, \cdots, \mu_d)$, $\cdots$, $\boldsymbol{X}_d=(\mu_1, \mu_2, \cdots, \mu_d+f\sigma_d)$, $\boldsymbol{X}_{d+1}=(\mu_1-f\sigma_1, \mu_2, \cdots, \mu_d)$, $\cdots$, $\boldsymbol{X}_{d+j}=(\mu_1, \mu_2, \cdots, \mu_j-f\sigma_j, \cdots, \mu_d)$, $\cdots$, $\boldsymbol{X}_{2d}=(\mu_1, \mu_2, \cdots, \mu_d-f\sigma_d)$ (now $f=3$) and eventually obtain $2d+1$ groups of random samples $\boldsymbol{X}_i$ ($i=1, 2, \cdots, 2d+1$). Substitute each group into 3D M-P method to calculate the factor of safety of stability for the three-dimensional slope F_{3S}. Letting $F_{3S}-1=y_i$, we have $2d+1$ groups of data pair $(\boldsymbol{X}_i, y_i)$.

To determine the parameters $\alpha_i-\alpha_i^*, \gamma_i, b$ ($i=1, 2, \cdots, n$) in Eq.(16), the fittings for the $2d+1$ groups of sample data pair $(\boldsymbol{X}_i, y_i)$ are carried out on the basis of the SVM data fitting method in Eq.(16). Then, we get the fitting expression of structural performance function of stability for the three-dimensional slope based on SVM as

$$g^{(0)}(\boldsymbol{x})=F_{3S}(x_1, x_2, \cdots, x_d)-1=\sum_{i=1}^{2d+1}\left(\alpha_i-\alpha_i^*\right)\exp\left(-\gamma_i\left|\boldsymbol{X}-\boldsymbol{X}_i\right|^2\right)+b \tag{17}$$

where the superscript "(0)" indicates the first fitting.

Using design points to solve Eq.(17), reliability index $\beta^{(0)}$ and the corresponding design point $\boldsymbol{X}^{*(0)}$ can be obtained. Then, letting $\boldsymbol{X}^{*(0)}$ be the datum point $\boldsymbol{X}_0$ and $f=1$, we update the $2d+1$ groups of random samples and repeat the prescribed calculation again.

3.3 Solution steps of three-dimensional slope reliability

Assuming that random variables x_i ($i=1, 2, \cdots, n$) which influence the stability of the three-dimensional slope are normally distributed (other distribution forms can be transformed to the normal distribution in equivalent), solution steps to obtain the reliability index for the three-dimensional slope based on SVM method are presented as follows:

1) Compute the performance function values at the present $2d+1$ groups of sample points by 3D M-P computation program;

2) Select the optimal supported vector from the $2d+1$ groups of performance function values by SVM program, and derive the relevant coefficients $\alpha_i-\alpha_i^*, \gamma_i$ and b ($i=1, 2, \cdots, n$) in Eq.(17), then yield the fitting performance function $g(\boldsymbol{x})$;

3) Apply design point method to solve structural reliability index $\beta^{(k)}$ and the corresponding design point $\boldsymbol{X}^{*(k)}$, in which the superscript k indicates the k-th iteration.

4) Check if it meets the convergence requirement, $\left|\beta^{(k)}-\beta^{(k+1)}\right|<\varepsilon$, where ε is the prescribed allowance error. If it does, stop the iteration; otherwise, select the corresponding design point and return to step 1) to iterate until the value of $\beta^{(k)}$ converges.

4 Case study

4.1 Case study of reliability for three-dimensional slope

The geometrical model of the slope adopts the case in Ref.[3]: knowing that the ratio of slope is 1:2, the height of slope is 12.2 m, the unit weight $\gamma=19.2$ kN/m^3, the mechanic strength indexes $c=29.3$kPa and $\varphi=20°$, similar to the original case, the sliding surface is an ellipsoid of revolution, so we can use an arc to simulate the sliding surface in the plane of symmetric axis and an ellipsoid surface in the direction of axis Z. Suppose that the equation of sliding surface is

$$\frac{(X-X_0)^2}{a^2}+\frac{(Y-Y_0)^2}{a^2}+\frac{Z^2}{b^2}=1$$

where X_0=5.102 m, Y_0=19.165 m, a=24.4 m and b=73.1 m. Uncertain factors are assumed to be c, φ and γ with variation coefficients as 0.2, 0.15 and 0.1, respectively, considering the likely variation range of each parameter.

The expression form of structural performance function of the three-dimensional slope can be represented as $g(\boldsymbol{x})=F_{3S}(c, \varphi, \gamma)-1$, and it can be solved by SVM method with a projected corresponding computation program. So, the reliability index can be obtained by steps as follows:

1) Take the mean value of initial iteration points, namely $\boldsymbol{X}_0$=(29.3, 20, 19.2) and let f=3 in the first sample of design point. Using the method in Section 3.2, we got seven sample points. After performing seven times of procedure computation for three-dimensional slopes, seven performance function values of sample points were obtained. Taking advantage of SVM method to select the optimal supported vector, with some relevant coefficients $\alpha_i-\alpha_i^*, \gamma_i$ and b which could be obtained in Eq.(17), we determined the fitting performance function and finished the first round of computation. The results of the first round of iteration by design point method are shown as: the reliability index $\beta^{(0)}$=3.491 6 and the corresponding design point $\boldsymbol{X}^{*(0)}$=(13.528 0, 13.789 3, 20.761 5).

2) Let $\boldsymbol{X}^{*(0)}$ be the iteration point and f =1, and repeat the previous calculation. The results are: the reliability index of the second round of iteration $\beta^{(2)}$=4.330 4, and the third one $\beta^{(3)}$=4.342 2. Since $|\beta^{(3)}-\beta^{(2)}|$=0.000 1<$\varepsilon$=0.005, the iteration is of convergence. After four rounds of iteration and 28 times of program calculation for the three-dimensional slope, we had the reliability index of stability for the three-dimensional slope, namely β=4.342 3, and the design point $\boldsymbol{X}^{*(3)}$=(10.233 1, 11.488 1, 20.129 0), in contrast to the result of Monte Carlo method in 100 000 times of sampling as β=4.291 8. Comparison among SVM method, response surface method and Monte Carlo method (results of Monte Carlo method have been widely accepted to be the exact solution) was performed, as listed in Table 1.

Table 1 Comparison among different methods

Method	Iteration times	Reliability index, β	Relative error/%	Relative workload/ %
Response surface	6	4.465 3	4.043	0.042
SVM herein	4	4.342 3	1.177	0.028
Monte-Carlo	100 000	4.291 8	—	—

It is shown that the workload of SVM method in reliability analysis for the three-dimensional slope decreases sharply compared with the exact method, but with corresponding computation accuracy. With respect to the fitting of complicated nonlinear performance functions, the method applied in this work is of superiority to response surface method.

4.2 Time-variant reliability for three-dimensional slopes

The weathering of rock-soil mass on the ground surface is chronic. It results in the increase of weathering depth over time, together with the varying of shear strength parameters. This can be defined that the shear strength parameters have time effect. In association with the case in Section 4.1, the expressions of values of shear strength parameters c and φ versus time are used: $c_i(t)=c_0(0.3525\text{e}^{-0.142\,6t}+0.6121\text{e}^{-0.003\,94t})$ and $\varphi_i(t)=\varphi_0\cdot(0.1822\cdot\text{e}^{-0.341\,5t}+0.824\,6\text{e}^{-0.002\,72t})$ (in this case, c_0=29.3 kPa and φ_0=20°). Assuming that the variation coefficient keeps constant, applying the calculation method of reliability referred in Section 4.1, letting t equal 3, 6, 10, 15, 20, 25, 30, 40, 50 a to compute the reliability value respectively, we obtain the results, as shown in Fig.4.

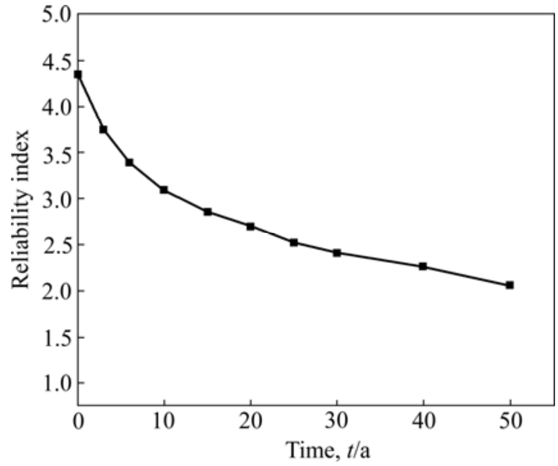

Fig.4 Reliability variation of three dimensional slopes versus time

From Fig.4, we can find that taking time effect of shear strength parameters of slope soils into account, the reliability index in 50 a descends by 52.54%, and the failure probability ascends from 0.000 705% to 1.966%, which indicates that methods that do not account for time effect will be unsafe when analyzing the reliability of slope. The varying of reliability for the three-dimensional slope deserves more attentions.

4.3 Discussion

4.3.1 Influence of combinations of variation coefficients of c and φ on reliability index

Since the variation range of variation coefficients of c and φ is larger than that of γ, we can fix the variation coefficient of γ as nought for simplicity, and change the values of variation coefficients of c and φ, respectively, to solve for the reliability index, as illustrated in Table 2 and Fig.5.

Table 2 Reliability value of different variation coefficients of c and φ

Variation coefficient of c, V_c	Variation coefficient of φ, V_φ				
	0.1	0.2	0.3	0.4	0.5
0.1	7.634 0	4.665 3	3.334 6	2.573 4	2.011 9
0.2	5.024 0	3.810 9	2.953 4	2.380 9	1.930 4
0.3	3.632 0	3.076 5	2.563 8	2.156 6	1.842 9
0.4	2.773 4	2.522 0	2.191 3	1.942 7	1.690 6
0.5	2.280 4	2.134 4	1.925 8	1.702 8	1.533 8

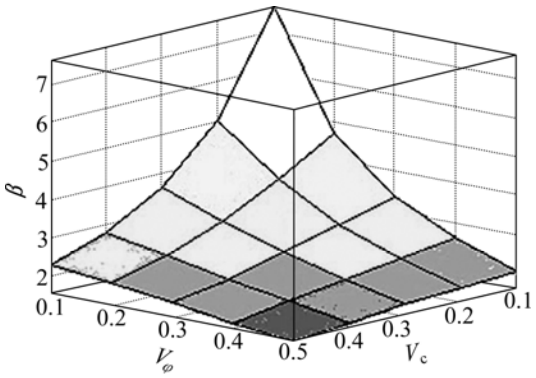

Fig.5 Three-dimensional surface diagram of β on different combinations of variation coefficients of c and φ

It can be concluded from Table 2 and Fig.5 that the effect of the varying of variation coefficient of φ is slightly larger than that of c, and the reliability index falls down rapidly with the increase of variation coefficients of c and φ. Accounting for the fact that the reliability value is magnificently affected by the magnitudes of variation coefficients of c and φ, it is necessarily required to get more accurate variation coefficients by tests as many as possible.

4.3.2 Time-variant reliability of three-dimensional slope on various variation coefficients

For brevity, values of c, φ and γ all equal 20, with an identical variation coefficient and the same amplitudes of variation. When changing the variation coefficient to be 0.05, 0.1, 0.15, and 0.2, respectively, adopting the case in Section 4.1 and the method in Section 4.2 again, we obtain the values of the time-variant reliability index for the three-dimensional slope. The results are shown in Table 3 (where $\beta_{0.05}$ denotes the value of reliability index for the three-dimensional slope when the variation coefficient equals 0.05, others by analogy) and Fig.6.

Table 3 and Fig.6 illustrate that the reliability index decreases gradually when the variation coefficient increases. The decrement descends gradually on the same increment of variation coefficient (all equal 0.05). Table 4 shows that the value of $\beta_{0.10}/(\beta_{0.05}-\beta_{0.10})$ is approximately equal to 1, $\beta_{0.15}/(\beta_{0.10}-\beta_{0.15})$ to 2 and $\beta_{0.20}/(\beta_{0.15}-\beta_{0.20})$ to 3. A specific regular pattern can be observed for the decrement of the reliability index.

Table 3 Time-variant reliability index for three-dimensional slope on different variation coefficients

Time/a	$\beta_{0.05}$	$\beta_{0.10}$	$\beta_{0.15}$	$\beta_{0.20}$
0	11.746	5.872	3.914	2.936
3	9.369	4.694	3.129	2.347
6	8.012	4.014	2.677	2.019
10	6.934	3.477	2.320	1.740
15	6.162	3.076	2.048	1.542
20	5.623	2.804	1.873	1.410
25	5.192	2.592	1.726	1.299
30	4.827	2.411	1.603	1.207
40	4.105	2.071	1.382	1.035
50	3.449	1.738	1.156	0.867

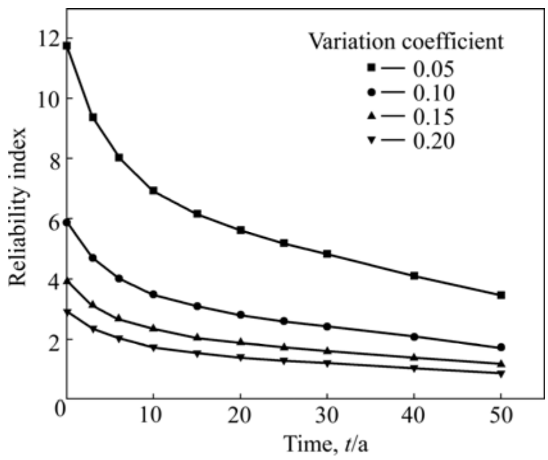

Fig.6 Time-variant reliability index for three-dimensional slope vs variation coefficients

Table 4 Variation of decrement amplitude of reliability index

Time/a	$\dfrac{\beta_{0.10}}{\beta_{0.05}-\beta_{0.10}}$	$\dfrac{\beta_{0.15}}{\beta_{0.10}-\beta_{0.15}}$	$\dfrac{\beta_{0.20}}{\beta_{0.15}-\beta_{0.20}}$
0	0.999 7	1.999 0	3.002 0
3	1.004 1	1.999 4	3.001 3
6	1.004 0	2.002 2	3.068 4
10	1.005 8	2.005 2	3.000 0
15	0.996 8	1.992 2	3.047 4
20	0.994 7	2.011 8	3.045 4
25	0.996 9	1.993 1	3.042 2
30	0.997 9	1.983 9	3.048 0
40	1.018 2	2.005 8	2.982 7
50	1.015 8	1.986 3	3.000 0

5 Conclusions

1) Based on 3D M-P method, applying SVM to approximate design point of highly nonlinear performance function, a new reliability analysis approach for the three-dimensional slope is developed.

2) Case study verifies that the time-consuming of reliability analyses approach based on small samples is 0.28% of that by accurate method when achieving corresponding accuracy that is superior to response surface method.

3) Analyses on time-variant reliability for three-dimensional slopes show that it is unsafe not to account for time effect and the time-variant of reliability cannot be ignored.

4) Values of variation coefficients of c and φ affect the reliability index largely and accurate variation coefficients are prerequisite to the reliability calculation. The decrement of the reliability index decreases gradually on the same increment of variation coefficient, which implies a specific regular pattern.

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